RelativeL-RittOrderOfEntireDirichletSeries

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Abstract

Recently Lahiri and Banerjee [1] have introduced the concept of Ritt-order of an entire Dirichlet Series and proved sum and product theorems. They as obtained Ritt order for derivatives. In this paper, we introduced the concept of L-Ritt order and discuss it for sum, products and derivatives of functions.

Keywords: Entire dirichlet series, Ritt order, relative L – Ritt order, property (A).

1. Introduction, Definition and Lemmas

For entire functionsg$_1$ and g$_2$ let $G_1(r) = \max\{|g_1(z)| : |z| = r\}$ and

$G_2(r)$ $\max\{|g_2(z)| : |z| = r\}$.

If g_1 is non constant then $G_1(r)$ is strictly increasing and a continuous function of r and its inverse G_1^{-1}: $(|g_1(0)|, \infty) \to (0, \infty)$ exits and $\lim_{R\to\infty} G_1^{-1}(R)=\infty$.(1.1)

Bernal [5] introduced the definition of relative order of g_1 with respect to g_2 denoted by $\rho_{g_2}(g_1)$ as follows

$\rho_{g_2}(g_1) = \inf\{\mu > 0 : G_1(r) < G_2(r^{\mu}) forallr > r_0(\mu) > 0\}$.(1.2)

Let $f(s)$ be an entire function of the complex variable $s = \sigma + it$ defined by everywhere absolutely convergent Dirichlet series $\sum_{n=1}^{\infty} a_n e^{s\lambda_n}$ (1.3)

where $0 < \lambda_n < \lambda_{n+1}(n \geq 1), \lambda_n \to \infty$ as $n \to \infty$ and $a_n^{,}s$ are complex constants.

If σ_c and σ_a denote respectively the abscissa of convergence and absolute convergence of (1.3) then in this case clearly $\sigma_c = \sigma_a = \infty$.

Let $F(\sigma) = \underset{-\infty < t < \infty}{l.u.b} |f(\sigma + it)|$ (1.4)

Then the Ritt order [16] of $f(s)$ denoted by $\rho(f)$ is given by

$$\rho(f) = \lim_{\sigma \to \infty} \sup \frac{\log \log F(\sigma)}{\sigma} = \lim_{\sigma \to \infty} \sup \frac{\log^{[2]} F(\sigma)}{\sigma}. \quad (1.5)$$

In other words $\rho(f) = \inf\{\mu > 0 : \log F(\sigma) < \exp(\sigma\mu) forall \sigma > R(\mu)\}$.(1.6)

Similarly the lower Ritt order of $f(s)$ denoted by $\lambda(f)$ may be defined.

In the paper we prove sum results on the related to relative L-Ritt order of an entire Dirichlet series. where $L = L(\sigma)$ is a positive continuous function increasing slowly i.e. $L(a\sigma) \approx L(\sigma)$ as $\sigma \to \infty$ for every constants a .In the paper we do not explain the standard definitions and notations in the theory of entire functions as those are available in [6].The following definitions are well known .

Definition 1. The relative Ritt order of $f(s)$with respect to an entire function $g(s)$ is defined by

$\rho_g(f) = \inf\{\mu > 0 : \log F(\sigma) < G(\sigma\mu)$ For all large σ }(1.7)

where $G(r) = \max\{|g(s)| : |s| = r\}$. Clearly $\rho_g(f) = \rho(f)$ if $g(s) = e^s$. The following analogous definition from [5] will be needed.

Definition 2.A nonconstant entire function $g(s)$ is said to have the property (A) if for any $\delta > 1$ and positive $\sigma, [G(\sigma)]^2 \leq G(\sigma^{\delta})$ holds where $G(\sigma) = \max\{|g(s)| : |s| = \sigma\}$.

Definition 3. The L-Ritt order $\rho_f^L \equiv \rho^L(f)$ and the L-Ritt lower order $\lambda_f^L \equiv \lambda^L(f)$of $f(s)$ are defined as follows respectively

$$\rho^L(f) = \lim_{\sigma\to\infty}\sup \frac{\log^{[2]} F(\sigma)}{\sigma L(\sigma)} \quad (1.8) \qquad \lambda^L(f) = \lim_{\sigma\to\infty}\inf \frac{\log^{[2]} F(\sigma)}{\sigma L(\sigma)} \quad (1.9)$$

Where $\log^{[k]} x = \log(\log^{[k-1]} x)$ for k=1,2, 3 , … and $\log^{[0]} x = x$. Similarly one can define the relative L-Ritt and relative lower L-Ritt order of $f(s)$

Definition 4. The relative L-Ritt order $\rho_g^L(f)$ and the relative lower L-Ritt order $\lambda_g^L(f)$ of $f(s)$ with respect to entire $g(s)$ are respectively defined as

$$\rho_g^L(f) = \lim_{\sigma\to\infty}\sup \frac{G^{-1}\log F(\sigma)}{\sigma L(\sigma)} \quad (1.10) \qquad \lambda_g^L(f) = \lim_{\sigma\to\infty}\inf \frac{G^{-1}\log F(\sigma)}{\sigma L(\sigma)} \quad (1.11)$$

Bernal [5] has proved the following.

Lemma1 [5]. If α >1, 0 < β <α then $G(\alpha\sigma) > \beta G(\sigma)$ for all large σ.

Lemma2 [5]. If g is transcendental with $g(0) = 0$ then for all large σ and $0 < \delta < 1$.

$$G(\sigma^{\delta}) < \bar{G}(\sigma) < G(2\sigma) \text{where } \bar{G}(\sigma) = \max\{|g'(z)| : |z| = \sigma\}$$

After Bernal , several papers on relative order of entire functions have appeared in the literature where growing interest of researcher on this topic has been noticed (see for example [2],[3],[4],[12],[13],[14], [15],[16]). During the past decades, several authors (see for example [17], [18], [20]) made close investigation on the properties of entire Dirichlet series related to Ritt order.

2.Main Results: Following the sections 1, have proved the following theorem.

Theorem 1. (a) $\rho_g^L(f) = \lim_{\sigma\to\infty}\sup \frac{G^{-1}\log F(\sigma)}{\sigma L(\sigma)}$.

(b) If $F_1(\sigma) \le F_2(\sigma)$ for all large σ, then $\rho_g^L(f_1) \le \rho_g^L(f_2)$.

Proof: (a) If $\varepsilon > 0$ is arbitrary then from the definition.

$$\rho_g^L(f)+\varepsilon > \frac{G^{-1}\log F(\sigma)}{\sigma L(\sigma)} \quad \text{forall large } \sigma \quad (2.1)$$

and there exist a sequence of value $\sigma = \sigma_n$ tending to infinity.

$$\frac{G^{-1}\log F(\sigma_n)}{\sigma_n L(\sigma_n)} > \rho_g^L(f)-\varepsilon$$ (2.2)From (2.1) and (2.2)

$$\limsup_{\sigma\to\infty} \frac{G^{-1}\log F(\sigma)}{\sigma L(\sigma)} = \rho_g^L(f).$$

Proof:(b) For arbitrary $\varepsilon > 0$ and for all large σ, we can write from (a).

$$F_2(\sigma) < \exp\left[G\left\{\sigma L(\sigma)\left(\rho_g^L(f_2)+\varepsilon\right)\right\}\right]$$

Since $F_1(\sigma) \le F_2(\sigma)$ for all large σ, we obtain

$$\rho_g^L(f_1) = \lim_{\sigma\to\infty} \frac{G^{-1}\log F_1(\sigma)}{\sigma L(\sigma)} \le \rho_g^L(f_2)+\varepsilon$$

Since $\varepsilon > 0$ is arbitrary $\rho_g^L(f_1) \le \rho_g^L(f_2)$.

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2.1Sum and Product Theorems

In this section, we assume that $f_{1,} f_2$ etc. are entire functions of s defined by everywhere absolutely convergent ordinary Dirichlet series $\sum_{n=1}^{\infty} \frac{a_n}{n^s}$, $\sum_{n=1}^{\infty} \frac{b_n}{n^s}$ etc. The product of two such series is considered by Dirichlet product method, which is also everywhere absolutely convergent (see [9].pp 66).

Theorem 2. Let $g(s)$be an entire function having the property (A).Then

(i) $\rho_g^L(f_1 \pm f_2) \le \max\left\{\rho_g^L(f_1), \rho_g^L(f_2)\right\}$ Sign of equality holds when $\rho_g^L(f_1) \ne \rho_g^L(f_2)$

and (ii) $\rho_g^L(f_1 f_2) \le \max\left\{\rho_g^L(f_1)\rho_g^L(f_2)\right\}$.

Proof: (i) We may suppose that $\rho_g^L(f_1)$ and $\rho_g^L(f_2)$ both are finite, because in the contrary case the inequality follows immediately .We prove (i) for addition only, because the proof for

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subtraction is analogous.

Let $f = f_1 + f_2$, $\rho = \rho_g^L(f)$ $\rho_i^L = \rho_g^L(f_i)$ $i = 1,2$ and $\rho_g^L(f_1) \le \rho_g^L(f_2)$.

For arbitrary $\varepsilon > 0$ and for all large σ, we have from Theorem 1(a)

$$F_1(\sigma) < \exp[G(\sigma L(\sigma)(\rho_g^L(f_1)+\varepsilon))]$$

$$\le \exp[G(\sigma L(\sigma)(\rho_g^L(f_2)+\varepsilon))]$$

and $F_2(\sigma) < \exp[G(\sigma L(\sigma))(\rho_g^L(f_2)+\varepsilon)]$So for all large σ $F(\sigma) \le F_1(\sigma) + F_2(\sigma)$

$$\le 2\exp[G(\sigma L(\sigma)(\rho_g^L(f_2)+\varepsilon))]$$

$< \exp[G(\sigma L(\sigma)(\rho_g^L(f_2)+\varepsilon))]^2$, since for all $x, 2\exp(x) < \exp(x^2)$

$\le \exp[G(\sigma L(\sigma)(\rho_g^L(f_2)+\varepsilon))]^\delta$ For every $\delta > 1$,by property (A).Therefore

$\dfrac{G^{-1}\log F(\sigma)}{\sigma L(\sigma)} < \{\rho_g^L(f_2)+\varepsilon\}^\delta \sigma^{\delta-1}\{L(\sigma)\}^{\delta-1}$ for all large σ

Taking first $\delta \to 1+0$ and then limit superior as $\sigma \to \infty$ and nothing that $\varepsilon > 0$ is arbiyrary, we obtain $\rho_g^L(f) < \rho_g^L(f_2)$. This proves the first part of(i).

For the second part of (i), let $\rho_g^L(f_1) < \rho_g^L(f_2)$.

andsuppose that $\rho_g^L(f_1) < \mu < \lambda < \rho_g^L(f_2)$.

Then for all large σ $F_1(\sigma) < \exp[G(\sigma L(\sigma)\mu)]$(2.3)

and there exist an increasing sequence $\{\sigma_n\}, \sigma_n \to \infty$

$F_2(\sigma_n) > \exp[G(\sigma_n L(\sigma_n)\lambda)]$for $n = 1,2,3,...$ (2.4)

Using Lemma 1, by setting $\alpha = \dfrac{\lambda}{\mu}$, $r = \sigma\mu$, $\beta = 1+\varepsilon$, $0 < \varepsilon < 1$ such that $1 < \beta < \alpha$, we obtain

$$G\left(\frac{\lambda}{\mu}\sigma\mu\right) > (1+\varepsilon)G(\sigma\mu)$$

i.e. $G(\lambda\sigma) > (1+\varepsilon)G(\sigma\mu)$

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Therefore using (2.3) and (2.4) and the fact that $G(\sigma) > \frac{\log 2}{\varepsilon}$ for all large σ, we obtain

$$F_2(\sigma_n) > \exp[G(\sigma_n L(\sigma_n).\lambda)]$$

$$> \exp[(1+\varepsilon)G(\sigma_n L(\sigma_n).\mu)]$$

$$> 2\exp[G(\sigma_n L(\sigma_n).\mu)]$$

$> 2F_1(\sigma_n)$, for all large n. (2.5)

Now $$F(\sigma_n) \geq F_2(\sigma_n) - F_1(\sigma_n)$$

$> F_2(\sigma_n) - \frac{1}{2}F_2(\sigma_n)$, using (2.5)

$$= \frac{1}{2}F_2(\sigma_n)$$

$> \frac{1}{2}\exp[G(\sigma_n L(\sigma_n).\lambda)]$, from (2.4)

$> \exp[(1-\varepsilon)G(\sigma_n L(\sigma_n).\lambda)]$, for all large n.

Let $\rho_g^L(f_1) < \lambda_1 < \lambda < \rho_g^L(f_2)$, and $0 < \varepsilon < \frac{\lambda - \lambda_1}{\lambda}$ (which is clearly permissible).

Using Lemma 1, by setting $\alpha = \frac{\lambda}{\lambda_1}$, $\beta = \frac{1}{1-\varepsilon}$, $r = \sigma\lambda_1$, we have, because $0 < \beta < \alpha$

$$G\left(\frac{\lambda}{\lambda_1}\sigma\lambda_1\right) > \frac{1}{1-\varepsilon}G(\sigma\lambda_1),$$

i.e. $(1-\varepsilon)G(\lambda\sigma) > G(\sigma\lambda_1)$.

Hence for all large n, $F(\sigma_n) > \exp[G(\sigma_n L(\sigma_n).\lambda_1)]$,

i.e. $\frac{G^{-1}\log F(\sigma_n)}{\sigma_n L(\sigma_n)} > \lambda_1$ for all large n.

This gives $\rho_g^L(f) \geq \lambda_1$. Since λ & λ_1 both are arbitrary in the interval $(\rho_g^L(f_1), \rho_g^L(f_2))$,

We have $\rho_g^L(f) \geq \rho_g^L(f_2) = \max\{\rho_g^L(f_1), \rho_g^L(f_2)\}$, i.e. $\rho_g^L(f_1+f_2)\geq \max\{\rho_g^L(f_1),\rho_g^L(f_2)\}$.

This in conjunction with the first part of (i) gives

$$\rho_g^L(f_1+f_2)=\max\{\rho_g^L(f_1),\rho_g^L(f_2)\}$$

which proves (i) completely.

(ii) Let $f=f_1f_2$ and the notations $\rho_g^L(f),\rho_g^L(f_1)$ and $\rho_g^L(f_2)$ have the analogous meanings as in (i). If $\rho_g^L(f_1)\leq\rho_g^L(f_2)$ then for arbitrary $\varepsilon>0$ for all large σ

$$F(\sigma)\leq F_1(\sigma).F_2(\sigma)$$

$$<\exp[G(\sigma L(\sigma)(\rho_g^L(f_1)+\varepsilon))].\exp[G(\sigma L(\sigma)(\rho_g^L(f_2)+\varepsilon))]$$

$$\leq\exp[2G(\sigma L(\sigma)(\rho_g^L(f_2)+\varepsilon))]$$

$$\leq\exp[G(\sigma L(\sigma)(\rho_g^L(f_2)+\varepsilon))]^2$$

$\leq\exp\left[G\{\sigma L(\sigma)(\rho_g^L(f_2)+\varepsilon)\}^{\delta}\right]$ for every $\delta>1$,by property (A).

The above gives $\dfrac{G^{-1}\log F(\sigma)}{\sigma L(\sigma)}\leq(\rho_g^L(f_2)+\varepsilon)^{\delta}\sigma^{\delta-1}(L(\sigma))^{\delta-1}$ for all large σ .Letting $\delta\to 1+0$ and then considering the fact that $\varepsilon>0$ is arbitrary, we obtain $\rho_g^L(f)\leq\rho_g^L(f_2)$ which proves the theorem.

2.2 Relative L-Ritt order of the derivative

Theorem 3. Let $f(s)$ be an entire function defined by the Dirichlet series (1) having finite L-Ritt order $\rho^L(f)$ and $f'(s)$ be its derivative .Then $\rho_g^L(f)=\rho_g^L(f')$where g(s) is a transcendental entire function.

Proof: It is known([17], p139) that for all large value of σ and arbitrary $\varepsilon>0$

$F(\sigma)-\varepsilon<(\sigma L(\sigma)-\sigma_0L(\sigma_0))F'(\sigma)+|f(s_0)|$ (3.1)

where $s_0=\sigma_0+it_0$ is a fixed complex number and $F'(\sigma)=\underset{-\infty<t<\infty}{l.u.b.}|f'(\sigma+it)|$.

The inequality (3.1) implies $F(\sigma)<(\sigma L(\sigma))F'(\sigma)+A+\varepsilon$, where A is a constant. Taking logarithm, we see that for all large value of σ

$\log F(\sigma) < \log[(\sigma L(\sigma))F'(\sigma)] + B_\sigma$ Where $B_\sigma \to \infty$ as $\sigma \to \infty$

$< \log F'(\sigma) + \log(\sigma L(\sigma)) + B_\sigma$

$< \log F'(\sigma) + \sigma L(\sigma)(\rho_g^L(f') + \varepsilon) + B_\sigma$

$< \log F'(\sigma) + \sigma L(\sigma)(\rho_g^L(f') + 2\varepsilon)$

$< G[\sigma L(\sigma)(\rho_g^L(f') + \varepsilon)] + \sigma L(\sigma)(\rho_g^L(f') + 2\varepsilon)$

$< G[\sigma L(\sigma)(\rho_g^L(f') + 2\varepsilon)]$ (3.2)

because $\dfrac{G[(\sigma L(\sigma))(\rho_g^L(f') + \varepsilon)] + (\sigma L(\sigma))(\rho_g^L(f') + 2\varepsilon)}{G[(\sigma L(\sigma))(\rho_g^L(f') + 2\varepsilon)]} < 1$ for all large σ on using ([5],(d),p213) and ([6],p165).

From (3.2) $\rho_g^L(f) = \limsup_{\sigma\to\infty} \dfrac{G^{-1}\log F(\sigma)}{\sigma L(\sigma)} \le \rho_g^L(f') + 2\varepsilon$.

Since $\varepsilon > 0$ is arbitrary, $\rho_g^L(f) = \rho_g^L(f')$

To obtain the reverse inequality, we use the following inequality from ([17], p139)So

$F'(\sigma) - \varepsilon \le \dfrac{1}{\delta} F(\sigma + \delta)$ (3.3)

where $\varepsilon > 0$ is arbitrary and $\delta > 0$ is fixed.

So $\log F'(\sigma) \le \log\left(\dfrac{1}{\delta} F(\sigma+\delta) + \varepsilon\right)$

$= \log F(\sigma+\delta) + \log\left(\dfrac{1}{\delta} + \dfrac{\varepsilon}{F(\sigma+\delta)}\right)$

$\le G[(\sigma+\delta)L(\sigma+\delta)(\rho_g^L(f) + \varepsilon)] + \log\left(\dfrac{1}{\delta} + \dfrac{\varepsilon}{F(\sigma+\delta)}\right)$

$\le G[\{(\sigma+\delta)L(\sigma+\delta)\}(\rho_g^L(f) + 2\varepsilon)]$ for all large σ .

Therefore $\rho_g^L\left(f^{'}\right)=\lim_{\sigma\to\infty}\sup\dfrac{G^{-1}\log F^{'}(\sigma)}{\sigma L(\sigma)}\leq\rho_g^L(f)+2\varepsilon$.

Since $\varepsilon>0$ is arbitrary, $\rho_g^L\left(f^{'}\right)\leq\rho_g^L(f)$ which proves the theorem.

If we assume $g(0)=0$,a simpler proof of the following theorem may be provided which relates the L-Ritt order of f relative to g and to its derivative $g^{'}$.

Theorem 4. Let $f(s)$ be an entire function defined by the Dirichlet series (1) and $g(s)$ be an entire transcendental function with $g(0)$, then

$$\frac{1}{2}\rho_g^L(f)\leq\rho_{g^{'}}^L(f)\leq\rho_g^L(f).$$

Proof: Since $g(s)$ is transcendental with $g(0)=0$,we have by Lemma 2 for all large σ and $0<\delta<1$

$$G\left(\sigma^{\delta}\right)<\bar{G}(\sigma)<G(2\sigma),$$

where $\bar{G}(\sigma)=\max\left\{\left|g^{'}(s)\right|:|s|=\sigma\right\}$. By computations it follows that

$$\frac{1}{2}G^{-1}(\sigma)<\bar{G}^{-1}(\sigma)<\left[G^{-1}(\sigma)\right]^{\frac{1}{\delta}},$$

forall large σ. Therefore we can write for all large σ

$$\frac{1}{2}\frac{G^{-1}[\log(F(\sigma))]}{\sigma L(\sigma)}<\frac{\bar{G}^{-1}[\log(F(\sigma))]}{\sigma L(\sigma)}\leq\frac{\left\{G^{-1}[\log(F(\sigma))]\right\}^{\frac{1}{\delta}}}{\sigma L(\sigma)}$$

, since $\log F(\sigma)$ is increasing and tending to infinity as $\sigma\to\infty$ (see [8], [9]).Letting $\delta\to 1-0$, we obtain for all large σ

$$\frac{1}{2}\frac{G^{-1}\log F(\sigma)}{\sigma L(\sigma)}<\frac{\bar{G}^{-1}\log F(\sigma)}{\sigma L(\sigma)}\leq\frac{G^{-1}\log F(\sigma)}{\sigma L(\sigma)},$$

and this gives $\frac{1}{2}\rho_g^L(f)\leq\rho_{g^{'}}^L(f)\leq\rho_g^L(f)$.

which proves the theorem.

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